NUMERICAL SOLUTION OF THE PRESSURE DISTRIBUTION IN OIL RESERVOIR AND WELL

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ABSTRACT

Predicting the distribution of pressure in an oil reservoir is of fundamental importance for its evaluation and maintenance, since the pressure varies in space over time. In this paper, we considered a one-dimensional weakly compressible fluid flow in an oil reservoir. Numerical solutions were carried out to determine the pressure distribution for one phase using the explicit finite difference method using MATLAB. The results obtained show that the effectiveness of this method depends on the chosen time step and simulation time.
Keywords: *efficiency, explicit method, time step and simulation time, matlab, porous media, pressure and temperatures, well, oil and reservoir.*

INTRODUCTION

An oil reservoir is a subterranean porous medium containing hydrocarbons trapped either structurally or stratigraphically. Therefore, the description of fluid flow through such a medium is extremely complex compared to the flow through pipes or pipelines [1]. Unlike flow in pipes or channels, there are no defined flow paths in porous media, making it difficult to evaluate their capacity as a function of pressure. Due to the complex nature of multiphase flow, the nonlinearity of their constitutive equations, and the complexity of the reservoir, the search for analytical solutions to practical problems of fluid flow is impossible. Therefore, the only way to solve such models is to use numerical methods such as the finite difference method or the finite element method [2]. Nowadays, reservoir modeling has become a common tool in the field of petroleum engineering, which is widely used to solve various hydrodynamic problems associated with the production of oil and gas from reservoirs. For example, the authors of [3] developed an Embedded Discrete Fracture Model (EDFM) to model reservoir composition using a grid of corner points. Their model proved to be reliable based on the results obtained, and also compatible with various types of numerical solution schemes in existing simulators. Abdulla et al. [4] also used a multi-scale method to model and simulate two-phase flow in a reservoir using MRST. Shen et al. [5] used parallel computing techniques to model two-phase flow in naturally fractured reservoirs using dual porosity. Their numerical results have shown that their simulator is accurate and scalable compared to commercial software, and the numerical scheme is also efficient.

1. Research methodology

As stated earlier, the only means by which complex multiphase fluid flow models can be solved is through the use of numerical methods such as the finite difference method, the finite element method, and the finite volume method. In this regard, we carried out a comparative study of finite difference methods for solving a one-dimensional transport equation with a discontinuity in the initial boundary value. Research has shown that this method has the advantage in terms of faster time than other methods if the desired level of accuracy is required, as they can use larger time steps. Numerical solutions using the explicit direct difference method are provided in this article to mathematically predict the pressure distribution in an oil reservoir for a single-phase, one-dimensional, weakly compressible fluid flow. Providing numerical solutions to the fluid flow equations through reservoir simulation will help in creating effective reservoir monitoring and pressure maintenance plans . to improve the ultimate recovery from the target reservoir and other reservoir systems.

The development of reservoir simulators begins with the formulation of a finite difference model for the equations that govern fluid flow in porous media. These equations describe the physical processes of interest in the reservoir and are presented in the form of systems of equations (SE), which take into account the dynamic relationship between the liquid, the porous medium, and the flow conditions of the system [6]. These SEs are formulated in accordance with three basic physical principles, such as the continuity equation, Darcy's law, and the equation of state. Let us consider a horizontal, one-dimensional, slightly compressible fluid (oil) flow, and assume that the reservoir has two outer boundaries closed to the flow, but has an inner boundary in the form of a production well [7]. The general form of the equation for the flow of a single-phase fluid has the form:

$$\frac{\partial}{\partial x}\left(\beta_c \frac{k_x A_x}{\mu_0 B}\frac{\partial P}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\beta_c \frac{k_y A_y}{\mu_0 B}\frac{\partial P}{\partial y}\right)\Delta y+\frac{\partial}{\partial z}\left(\beta_c \frac{k_z A_z}{\mu_0 B}\frac{\partial P}{\partial z}\right)\Delta z+q_{sc}=\frac{V_b}{a_c}\frac{\partial}{\partial t}\left(\frac{\varphi}{B}\right), \tag{1}$$

where P - liquid potential, k_x - reservoir rock permeability in the x direction (Darcy), μ - fluid viscosity, ν_b - total block volume, φ - reservoir rock porosity, C_t - total compressibility of oil and rock, B - oil-bearing volume factor , β_c - transmission coefficient, a_c – volume conversion coefficient, q - oil consumption, Δx - grid length, A_x – grid cell area in x direction. The inverse finite-difference approximation to a slightly compressible flow leads to an explicit calculation procedure for the pressure of a new time level [8]. Therefore, it is used to find solutions by solving an equation involving both the current state of the system and at a later time.

2. Basic fluid flow equation

Since the equation that makes up the mathematical model of the reservoir is too complex to be solved analytically, finite difference approximation is used to bring the equation to a form that can be solved on a digital computer. The process involves discretization of the derivative with respect to space and time. From Equation (1), the reservoir linear system is assumed to be in the x direction , and the discretization is defined by the equation as:

$$\left(\beta_c \frac{k_x A_x}{\mu_i B_i \Delta x}\right)_{i+\frac{1}{2}}(p_{i+1}-p_i)-\left(\beta_c \frac{k_x A_x}{\mu_i B_i \Delta x}\right)_{i-\frac{1}{2}}(p_i-p_{i-1})+q_{sc}=\left(\frac{V_b}{a_c}\frac{\varphi}{B_i^0}c_t\right)_i\frac{\partial p_i}{\partial t} \tag{2}$$

Simplifying equation (2) by introducing transmissibility terms results in:

$$T_{ix_{i+1/2}}(p_{i+1}-p_i)-T_{ix_{i-1/2}}(p_i-p_{i-1})+q_{sc}=\left(\frac{V_b}{a_c}\frac{\varphi}{B_i^0}c_t\right)_i\frac{\partial p_i}{\partial t} \tag{3}$$

2.1. Direct difference approximations for the fluid flow equation

Direct difference approximation of the flow equation at the base time level for the first derivative on the right side of the equation. (3) is expressed as:

$$\frac{\partial P}{\partial t}=\frac{P_{i+1}^{n+1}-P_i^n}{\Delta t}, \tag{4}$$

Substituting equation (4) into equation (2) and (3) at the time level t^n leads to the following result:

$$\left(\beta_c \frac{k_x A_x}{\mu_i B_i \Delta x}\right)_{i+1/2}^n(p_{i+1}^n-p_i^n)-\left(\beta_c \frac{k_x A_x}{\mu_i B_i \Delta x}\right)_{i-1/2}^n(p_i^n-p_{i-1}^n)+q_{sc1}=\left(\frac{V_b}{a_c}\frac{\varphi c_t}{B_i^0\Delta t}\right)_i(p_i^{n+1}-p_i^n). \tag{5}$$

In terms of transmissibility,

$$T_{ix_{i+1/2}}(p_{i+1}^n-p_i^n)-T_{ix_{i-1/2}}(p_i^n-p_{i-1}^n)+q_{sc}=\left(\frac{V_b}{a_c}\frac{\varphi c_t}{B_i^0\Delta t}\right)_i(p_i^{n+1}-p_i^n) \tag{6}$$

2.2 . Explicit Composition Calculation Method

$$p_{i+1}^n=p_i^n+\left(\frac{a_c B_i^0}{V_b}\frac{\Delta t}{\varphi c_t}\right)_i q_{isci}+\left(\frac{a_c B_i^0}{V_b}\frac{\Delta t}{\varphi c_t}\right)_i\times[T_{ix_{i+1/2}}^n p_{i+1}^n(T_{ix_{i+1/2}}^n+T_{ix_{i-1/2}}^n)p_i^n+T_{ix_{i-1/2}}^n p_{i-1}^n] \tag{6'}$$

2.3. Inverse Difference Approximations for the Fluid Flow Equation

The inverse difference approximation is commonly used in reservoir modeling because its use does not limit the time step size for a stable solution. An approximation of the inverse difference of the first derivative at the base time level is written as:

$$\frac{\partial P}{\partial t} = \frac{P(t^{n+1}) - P(t^{n})}{\Delta t}, \tag{7}$$

$$\frac{\partial P_i}{\partial t} = \frac{P_i^{n+1} - P_i^{n}}{\Delta t}. \tag{8}$$

$$\left(\beta_c \frac{k_x A_x}{\mu_i B_i \Delta x}\right)_{i+\frac{1}{2}} (p_{i+1}^{n+1} - p_i^{n+1}) - \left(\beta_c \frac{k_x A_x}{\mu_i B_i \Delta x}\right)_{i-\frac{1}{2}} (p_i^{n+1} - p_{i-1}^{n+1}) + q_{sc1} = \left(\frac{V_b}{a_c} \frac{\varphi c_t}{B_i^0 \Delta t}\right)_i (p_i^{n+1} - p_i^{n}). \tag{9}$$

$$T_{ix_{i+1/2}} (p_{i+1}^{n+1} - p_i^{n+1}) - T_{ix_{i-1/2}} (p_i^{n+1} - p_{i-1}^{n+1}) + q_{sc1} = \left(\frac{V_b}{a_c} \frac{\varphi c_t}{B_i^0 \Delta t}\right)_i (p_i^{n+1} - p_i^{n}) \tag{10}$$

3. Implicit Calculation Method

The forward-difference approximation to the equation of flow of a weakly compressible fluid leads to an implicit procedure for calculating pressures at a new time level. Equation transformation. (10) gives:

$$T_{ix_{i+1/2}} p_{i+1}^{n+1} - \left[\left(\frac{V_b}{a_c B_i^0} \frac{\varphi c_t}{\Delta t}\right) + T_{ix_{i+1/2}} + T_{ix_{i-1/2}}\right] p_i^{n+1} + T_{ix_{i-1/2}} p_{i-1}^{n+1} = -\left[q_{isci} + \left(\frac{V_b}{a_c B_i^0} \frac{\varphi c_t}{\Delta t}\right)_i p_i^{n}\right]. \tag{10'}$$

where quantities P_{i+1}^{n+1}, P_i^{n+1} and P_{i-1}^{n+1} are all unknowns. In contrast to the explicit formulation, the equation. (10) cannot be solved explicitly for, P_i^{n+1} since P_{i+1}^{n+1} both P_{i-1}^{n+1} also unknown. As a result, equation (10) written for all grid blocks and unknowns must be solved simultaneously or using iterative algorithms. Using the last solution and after some algebraic operations, we get the original equation

$$\frac{\partial}{\partial x}\left(\frac{\beta_c A_x k_x}{\mu B} \frac{\partial P}{\partial x}\right) \Delta x + q_{sc} = \frac{v_b}{a_c}\left(\frac{\phi}{B}\right) C_t \frac{\partial P}{\partial t}. \tag{11}$$

(1) for the flow of a single-phase liquid in a tank. Note that a detailed procedure of the forward finite difference approximation for calculating the pressure of an easily compressible flow is given in [8]. Therefore, it is used to find solutions by solving an equation involving both the current state of the system and at a later time.

4. Implicit Backward Difference Method

In Figures 1-4 has been shown the simulation results obtained for the implicit method using a time step size of 1 day. In this case, the pressure may move more than one grid block in each time step. This is clearly seen in Fig. 1, since the pressure drop was observed not only on the grid block 3 on the 1st day, as in the explicit method, but also a pressure drop was observed on the block 3 and 5 for the simulation time of 1 day. This is because the implicit method requires solving unknown pressures for a pair of systems of equations. The implicit method showed the same smooth decrease in pressure first on neighboring blocks due to their proximity to the production well (grid block 4), and then moved to neighboring blocks, thereby confirming the statement that the closer the grid block is to the production well, the higher the pressure drop when withdrawing fluid. It may be the short time it takes for transient pressure to reach these grid blocks adjacent to the production well. Accordingly, grid boxes 3 and 5 show the same trend (constant pressure values) as in the explicit simulation results, but it took longer simulation time before the corresponding pressure values changed slightly and steadily. For grid blocks 2 and 6, the same pressure values were recorded from the 1st to the 55th day, then a decrease was observed from the 56th day. The ideal trend that was observed could be due to their placement side by side with grid blocks 3 and 5 respectively, so there is less pressure disturbance due to fluid intake. However, the change in their respective pressure values can be attributed to pressure perturbations occurring in nearby grid blocks (blocks 1 and 6) due to fluid withdrawal. In comparison, grid boxes 1 and 6 recorded the same trend (constant pressure values) as grid boxes 2 and 6 from the start of the simulation to the decline on day 35.

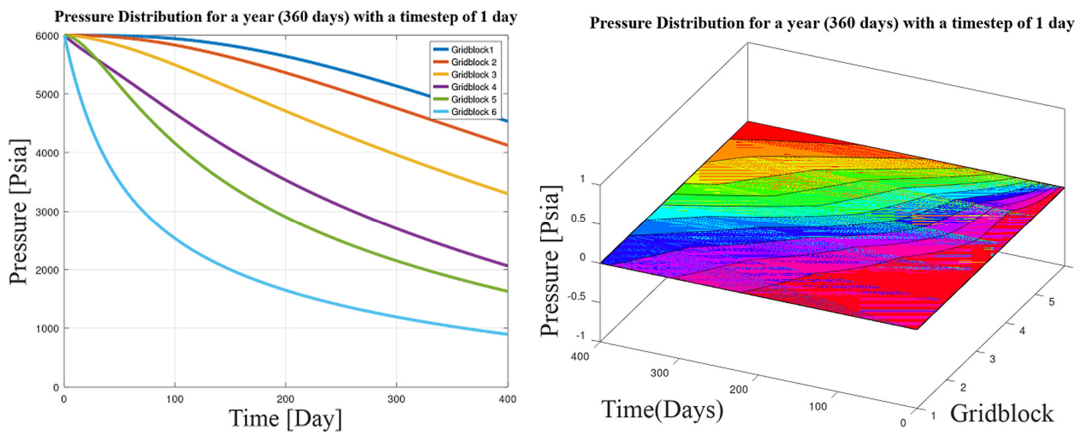

Figure 1 . Pressure distribution for a year with a timestep of 1 day using implicit method

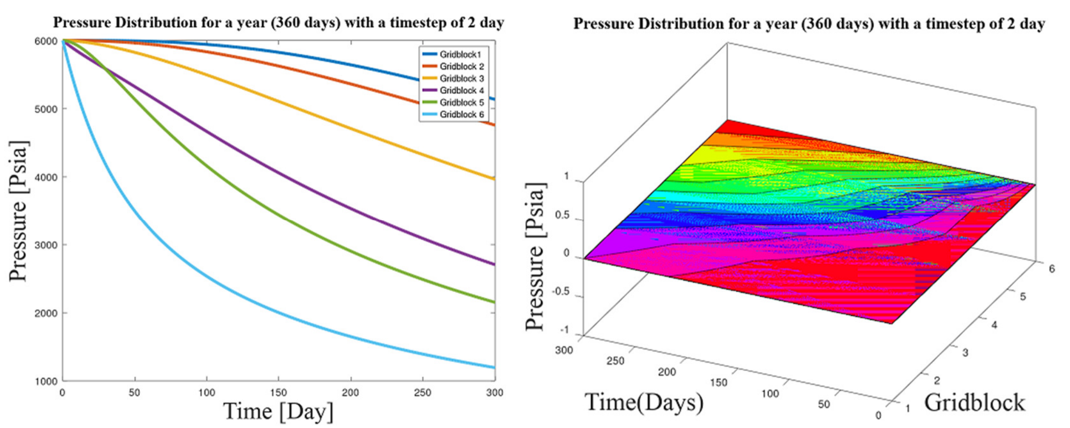

Figure 2. Pressure distribution for a year with a timestep of 2 days using implicit method

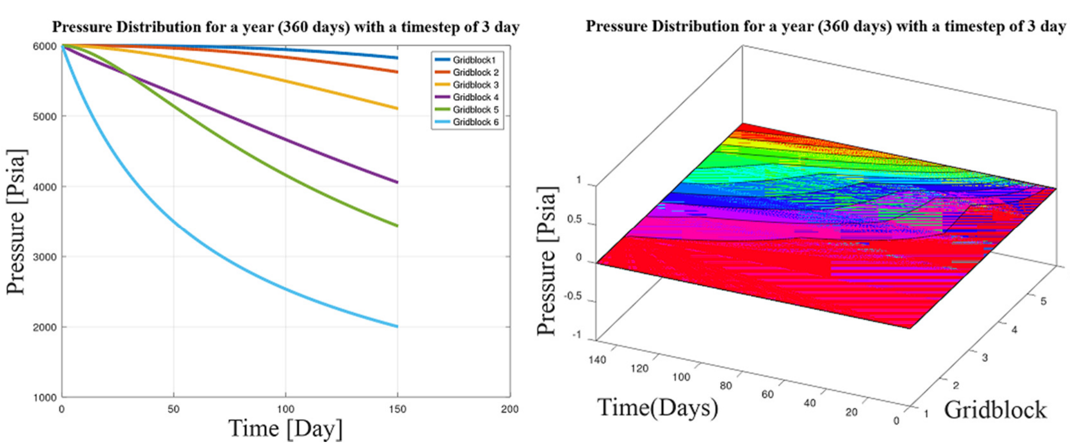

Figure 3. Pressure distribution for a year with a timestep of 3 days using implicit method

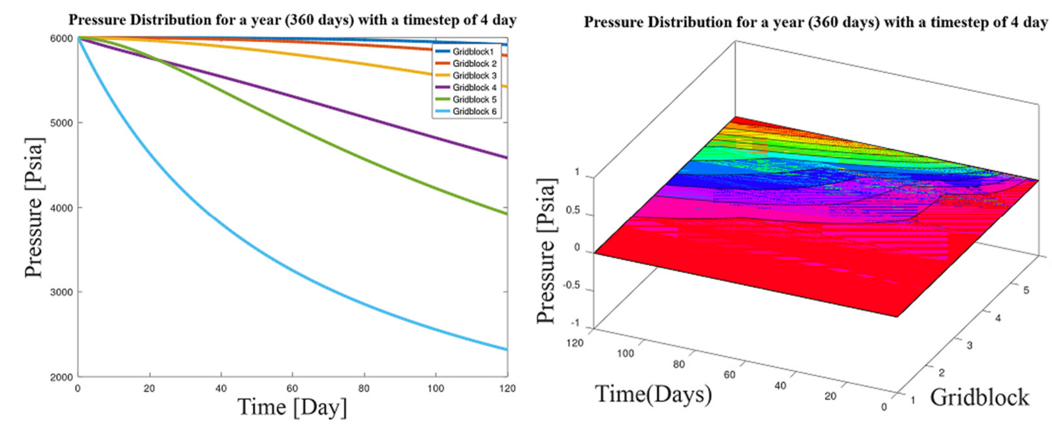

Figure 4. Pressure distribution for a year with a timestep of 4 days using implicit method

CONCLUSION

The simulation results comparing different time intervals showed that the stability of the explicit finite difference formulation depends on the specific conditions, the chosen time step, the reservoir geometry or the allocated simulation time. This is the main limitation of the explicit direct difference formulation, which reduces its use in oil reservoir modeling.

LITERATURE

1. Ahmed, T., McKinney, P.D., 2005. Advanced Reservoir Engineering , Gulf Professional Publishing, USA.
2. Chen, Z., Huan, G., Ma, Y. 2006. Computational Methods for Multiphase Flows in Porous Media. Computational Science and Engineering Series , Vol. 2, SIAM, Philadelphia, PA, USA.
3. Sun, X., Sun, M., Takabatake, K., Pain, C.C., Sakai, M. 2019. Numerical Simulation of Free Surface Fluid Flows through Porous Media by Using the Explicit MPS Method. Transport in Porous Media , Vol. 127, no. 1, pp. 7-33.
4. Abdullah, M.A., Panda, A., Gupta, S., Joshi, S., Singh, A., Rao, N. 2019. Multi-scale method for modeling and simulation of two phase flow in reservoir using MRST, Petroleum and Coal , Vol. 61, no. 3, pp. 546 - 558.
5. Shen, L., Cui, T., Liu, H., Zhu, Z., Zhong, H., Chen, Z., Yang, B., He, R., Liu, H. 2019. Numerical simulation of two-phase flow in naturally fractured reservoirs using dual porosity method on parallel computers: numerical simulation of two-phase flow in naturally fractured reservoirs. 2019 Proceedings of the International Conference on High Performance Computing in Asia-Pacific Region , Guangzhou, China.
6. Aphu, E.S., Brantson, E.T., Addo, B.J., Akunda, D., 2018. Development of finite difference explicit and implicit numerical reservoir simulator for modeling single phase flow in porous media, Earth Sciences, Vol . 7, no. 6, pp. 242-259.
7. Ertekin, T., Abou-Kassem, J.H., and King, G.R. 2007, Basic Applied Reservoir Simulation , SPE Textbook Series, Volume 10, Richardson, Texas, USA. 2007.
8. Prince O.A., Ohenewaa K.D., Eric T.B. A comparative study between finite difference explicit and implicit method for predicting pressure distribution in a petroleum reservoir International Journal of Engineering, Science and Technology Vol. 11, no. 4, 2019, pp. 23-40